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Wright Brothers Lectureship in Aeronautics

Missile Aerodynamics — Past, Present, Future

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Foreword

I AM deeply appreciative of the honor of being invited to deliver the Wright Brothers Lectureship in Aeronautics for 1979 in memory of those ingenious aeronautical pioneers who first reduced sustained powered aerial flight to practice. This is the first Wright Brothers lecture devoted to the subject of missiles. While any connection between missile aerodynamics and the Wright Brothers may not be obvious at first glance, there are several important connections. The use of the airplane for the aerial delivery of missiles accounts to a large degree for the extensive current importance of missiles and the engineering and scientific efforts which have gone into the advancement of missile aerodynamics since World War II. Also there is much commonality in the fields of missile aerodynamics and airplane aerodynamics, and each field has benefited from new developments in the other.

Introduction

My purposes are to give visibility to the aeronautical community generally of the importance of missile aerodynamics and to suggest specific areas which need future attention. An historical approach to the subject has been found convenient.

The principal problems of missile aerodynamics which will be addressed concern the aerodynamic characteristics of such configurations as bodies, wings, wing-body combinations, and wing-body-tail combinations from subsonic speeds to supersonic speeds. The aerodynamic advances in connection with these configurations will be traced from the past to the present, particularly with regard to our ability to predict their aerodynamic characteristics. Both engineering methods and theoretical methods will be discussed. The theoretical methods of interest include slender-body theory, supersonic linear theory, vortex methods, and the Euler equations. Linear methods are discussed as well as nonlinear methods which attempt to account for the effects of compressibility and viscosity. The paper concludes with a prognostication of important future problems.

In an account of missile aerodynamics, one can go back to prehistoric times. In an interesting history of missiles, Spearman[1] has traced the development of modern missiles and rockets from such ancient devices as spears, slings, arrows, and stones. The unguided ground-to-ground missile is thus the oldest missile type.

With respect to guided missiles during World War I the Dayton-Wright Airplane Co. obtained a contract from the Army Signal Corp. to do the aerodynamic design of a guided air-to-ground missile known as the Kettering Aerial Torpedo or the "Kettering Bug." Orville Wright, who had no stock interest in the company, was retained by it as an aerodynamic consultant on the aerial torpedo. Figure 1 depicts the aerial torpedo to be a biplane, a fact which should surprise nobody. The Bug was launched from a dolly running along a track. It was guided by a system of preset electric and vacuum-pneumatic controls. The engine was shut off after a predetermined time, the wings were released, and the Bug plunged to earth with 180 lb of explosives. The World War II V-1 missile was no different in concept.

Although successful flights of the Bug occurred in 1918, it was too late to be used in World War I. The Bug has been proclaimed by some to be the first guided missile. If that is so, then Orville Wright is the aerodynamic designer of the first guided missile. However, a competitive aerial torpedo was built by Lawrence Sperry and Glenn Curtis for the U.S. Navy at about the same time.

During World War II a number of efforts were carried on by the United States to develop guided missiles. This work included tests of heat-seeking glide bombs, television-guided missiles, and rocket-powered guided missiles. In Peenmunde, Germany, the missile "Wasserfall" was developed as sophisticated missile of the ground-to-air type incorporating advanced aerodynamic and guidance concepts.

Jack N. Nielsen is a recognized authority in missile aerodynamics and published a book on the subject in 1960. Dr. Nielsen received his B.S. in Mechanical Engineering from the University of California at Berkeley in 1941 and his Ph.D. in Aeronautics and Mathematics from the California Institute of Technology in 1951. Between 1941 and 1958 he was employed by NACA except for a period of duty with the 188th Engineering Combat Battalion during World War II. In 1958 he helped found VIDYA Inc. in Palo Alto, Calif., and in 1966 founded Nielsen Engineering & Research, Inc. Dr. Nielsen has served on a number of Government and AIAA committees, and has published about 160 papers. He is a fellow of the American Institute of Aeronautical Sciences and the Royal Aeronautical Society.

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In the United States extensive scientific development of all kinds of missiles was accelerated at the end of World War II. This development was motivated to a large extent by the success of German scientists in developing missiles during World War II. The extensive scientific development lasted until the successful launching of the Soviet satellite, Sputnik, after which many of the scientists and much of the monetary support were diverted to the space program. It is only in the last few years that missile aerodynamics has again become fashionable in the United States, even though many developments continued after 1958.

I am reminded of my first encounters with the aerodynamics of missiles. When young, I experimented with slings made of leather thongs and old shoe tongues. The accuracy of these devices was anything but satisfactory. I gave up slings with certain promptings from my father after several near catastrophies. In search of improved accuracy, I next took up the bow and arrow. The bows were made of native California wood "cured" by suspending them in the smoke of the chimney. The arrows were carved from board stock. Using glue made from the sap of trees, I affixed two chicken feathers to the aft end with indifferent results. Three feathers produced excellent results. Little did I realize I was experimenting with triform tail-body combinations!

The title of this paper divides the subject matter into three time periods: past, present, and future. I have arbitrarily divided the time into the three time periods shown in Fig. 2. The past is taken from the earliest time to Sputnik in 1958. The period from Sputnik until now is taken as the present. The future starts now.

I have remarked that there is currently much overlap between airplane and missile aerodynamics. What then are the differences between airplane and missile aerodynamics? Since missiles are not constrained in maneuverability by pilot limitations, they generally operate at higher normal accelerations than aircraft. Thus they operate at higher angles of attack than aircraft, usually well beyond the linear range. Operation at high angles of attack is accompanied by compressibility effects such as shock waves and viscous effects such as flow separation and vortices. Hence, contemporary missile aerodynamics has been concerned principally with nonlinear aerodynamic phenomena. In recent years the ability of aircraft to operate stably to high angles of attack in the transonic speed range at high altitudes has extended airplane aerodynamics into the angle-of-attack range of missile aerodynamics.

Several other significant differences between missiles and aircraft have been reflected in the subject matter of missile aerodynamics. Unlike aircraft, many missiles roll continuously in flight. Thus many missiles routinely utilize the entire range of bank angles, a fact leading to the use of cruciform lifting surfaces and circular bodies. In addition missiles are frequently more slender than airplanes, having bodies of higher fineness ratio and wings of lower aspect ratio. These differences have led to a greater emphasis on slender-body methods in missile aerodynamics. Because of the requirements of high maneuverability, fast response, and operation in the regime of aerodynamic nonlinearities, the problems of aerodynamic controls for missiles have been of special importance.

The present paper attempts to identify and describe many of the significant advances in aerodynamics of missiles: past, present, and future. The past and present are matters of record; the prediction of future trends will be less certain. However, I am interested in attempting such predictions in the hope they will encourage research in missile aerodynamics. It is clearly not appropriate nor indeed possible for the present paper to be a detailed review of all the important contributions to missile aerodynamics. Rather I shall attempt to discuss subject matter which, in my view, illustrates important trends. The emphasis will be on practice in the United States. I will not discuss strategic missiles.

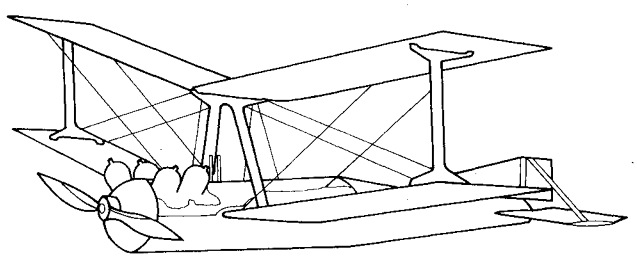


Fig. 1 Kettering aerial torpedo.

The Past

Developments before World War II

While missile aerodynamics did not exist as such in the U.S. prior to World War II, certain aerodynamic developments in this period were applicable to missiles. These developments include Munk's slender airship theory,[2] the body of revolution work of Von Kármán,[3,4] and the wing-tail interference work of Silverstein and Katzoff.[5]

It is interesting that one of the principal tools of missile aerodynamics should have its genesis in airship theory, namely, the slender airship theory of Max Munk.[2] Consider an airship passing through a plane fixed in the fluid and normal to the airship longitudinal axis. Since the airship is slender, the derivatives of the flow velocity component in the direction of the longitudinal axis are small compared to the derivatives of the components in planes normal to that axis. It is thus assumed that the flow in the cross-flow plane is an incompressible two-dimensional flow corresponding to the Laplace equation. The unsteady boundary conditions in the cross-flow plane correspond to a circle changing radius and

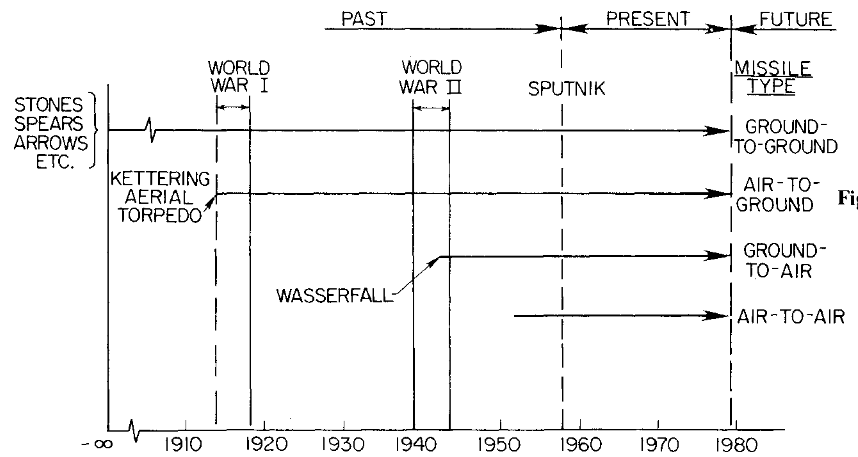


Fig. 2 Periods in missile development and missile types.

translating. This method makes it possible to calculate pressure distributions on any cross section for which the two-dimensional Laplace's equation can be solved, and thereby converts a three-dimensional flow problem into a two-dimensional one.

If only the axial loading distribution is needed, it is possible to avoid calculation of the flowfield by utilizing the powerful additional apparent-mass concept. The additional apparent mass of a cross section is that mass of fluid flowing at the cross-flow speed which would have the same kinetic energy per unit axial distance as the actual kinetic energy generated by the body per unit length. The additional apparent mass is a property of the cross-sectional shape, and tables exist for it for shapes such as those shown in Fig. 3. The method does not account for viscous effects.

Other advances in the aerodynamics of bodies of revolution in the pre-World War II period which are significant for missiles emerge from airship aerodynamics. In a paper first published in Germany[3] and later reprinted as an NACA technical memorandum,[4] Von Kármán used source and doublet distributions along the axes of bodies of revolution to calculate surface pressure distributions. This approach was later generalized for subsonic compressible flow using the Glavert-Prandtl rule[6] and to supersonic speed using supersonic source and doublets.[7,8] Von Kármán was further able to simplify his results for slender bodies and to derive the nose shape of minimum foredrag for a given length and a circular base of given diameter. The resulting shape is the well-known Kármán ogive. Using similar methods, Sears[9] and Haack[10] obtained the Sears-Haack body of minimum wave drag under the constraints of zero base area and a given length and volume.

The aerodynamics of wing-body interference as developed prior to World War II for airplanes were not applicable to missiles. Airplane practice was to consider the part of the wing blanketed by the fuselage to lift as if the fuselage were not there. This approach was acceptable for aircraft which had large wing spans relative to the fuselage diameter, but it is not accurate for missile configurations for which the ratio is much smaller. This problem was not solved until after World War II as will be discussed.

The interference between wing and tail due to operation of the tail in the downwash field of the wing was studied prior to World War II by Silverstein and Katzoff.[5] Their method, based on horseshoe vortex representations of the trailing vortex sheet, is applicable to missiles with planar wings and tails if the wake does not roll up.

Before and during World War II, German scientists and engineers undertook the systematic development of guided missiles on a large scale. In connection with missile aerodynamics, they studied such subjects as longitudinal stability from subsonic to supersonic speeds, missile controls, effects of roll on panel-panel interference and wing-tail interference, and optimum boat-tail drag. The importance of this work was recognized here and elsewhere, and a large-scale effort was launched in the United States to advance the state of the missile art. With regard to missile aerodynamics, the advances were particularly significant in the exploitation of slender-body theory, supersonic linear aerodynamic theory, and vortex theory.

Developments after World War II

It was toward the end of World War II that R. T. Jones resurrected the slender airship theory of Max Munk.[2] In a five-page paper[11] he showed how the method of Munk could be applied to low-aspect-ratio delta wings at speeds above and below the speed of sound. Thereafter a general exploitation of slender-wing theory and slender-body theory to all kinds of missile problems developed. The Ames Research Center of NACA was particlularly active in this effort as well as the subsequent development of linearized supersonic theory, vortex methods, and missile aerodynamic prediction methods. At the same time extensive development of theoretical and experimental missile aerodynamics was also started under the Bumblebee program of the U.S. Navy. References 12 and 13 give accounts of some of this work.

Exploitation of Slender-Body Theory

The slender-body developments sparked by the resurrection of the Munk airship theory needed a firm mathematical basis. This basis was supplied in a 1949 paper by Ward[14] in which he also applied the theory to determining lift and induced drag of slender planar wing-body combinations. Spreiter[15] in 1948 independently applied slender-body theory to determining the loadings of slender planar and cruciform wing-body combinations.

A particular advantage of the slender-body method is that any cross-sectional shape for which a cross-flow solution of Laplace's equation is known can be utilized in the method. For any missile with such cross-sectional shapes, the force and moment distributions can be calculated as well as the stability derivatives. All the power of conformal mapping methods and the theory of complex variables can be brought to bear on the subject in an elegant manner. The results are valid for slender missiles at low angles of attack well into the supersonic speed range.

Some specific results are of interest. For instance, Spreiter and Sacks[16] worked out the loading on circular bodies with cruciform wings as well as their wake characteristics. Sacks[17] applied slender-body theory to the calculation of the stability derivatives of slender bodies of general cross section. Bryson[18] applied conformal mapping to calculating forces, moments, and stability derivatives of a number of missile configurations.

With the use of cruciform wing missiles, the use of all-movable controls came into common practice with the canard fins or tail fins being rotated as a whole about some lateral hinge line. The forces and moments induced by various types

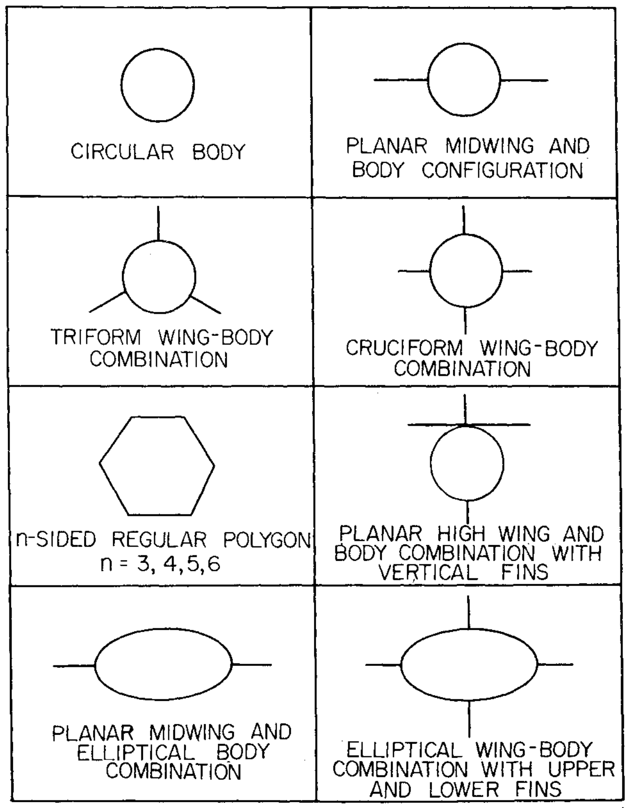


Fig. 3 Some cross-sectional shapes for which conformal mappings or apparent-mass solutions are known.

of control deflection were studied by such authors as Adams and Dugan[19] and Dugan and Hikido.[20]

In cases where only gross quantities were required, such as normal forces on wing or body or damping-in-roll derivatives, powerful reverse-flow theorems were derived by such investigators as Heaslet and Spreiter.[21] When these reverse-flow theorems are combined with slender-body theory, very useful results can be obtained easily.[22] It is also possible to extend these results to configurations which are not slender.

Supersonic Aerodynamic Theory

Prior to World War II the aerodynamic theory of supersonic flows was not well developed because there was little need for such theory. After World War II there was much doubt concerning the possibility of penetrating the sonic barrier with manned aircraft, although the V-2 rocket had already done so. When Chuck Yeager, in the X-1 airplane, broke the sonic barrier in 1947, there was already a substantial effort in supersonic aerodynamics underway in the U.S. The principal theoretical developments for bodies, wings, and wing-body combinations are now reviewed, particularly as they are applicable to missiles.

With regard to bodies, I have already mentioned that the solution for the supersonic flow for a pointed body of revolution at zero angle of attack was published by Von Kármán and Moore in 1932,[7] and the corresponding solution for angle of attack was published by Tsien[8] in 1938. The angle-of-attack problem was also solved by Ferrari[23] in 1937. These methods, using distribution of sources and doublets on the axis, will not produce bodies which have discontinuities in the axial derivative of the body radius (corners). Such discontinuities for slender bodies were studied by Lighthill in 1948.[24] Methods for computing the second-order supersonic pressure distributions past nonlifting bodies of revolution with corners or axial curvature discontinuities were developed by Van Dyke in 1952.[25] Thus while bodies of revolution at supersonic speeds had been treated before World War II, significant developments were made thereafter.

With regard to wings alone, it was generally possible to get only approximate solutions for their flowfields at subsonic speed before the use of high-speed computers. However, wings designed for supersonic flight tend to have sharp edges, and it was possible to obtain exact solutions for supersonic flow using linear aerodynamic theory. The supersonic wing flows were generally obtained by superimposing conical flows. A conical flow is one that has constant flow variables along each ray from some apex. Among the first work on this subject was the paper of R. T. Jones[26] which gave symmetrical wing thickness solutions from which wave drag could be calculated. For lifting surfaces, Busemann[27] gave the solution for rectangular wings at supersonic speeds.

The application of supersonic linear theory to a triangular wing with subsonic leading edges presented an important problem in supersonic aerodynamics for both missiles and aircraft. It is of interest that at least three investigators found the solution independently and at nearly the same time.[28-30] It is not infrequent that different workers independently make the same advance. Many investigations rapidly expanded the results to a wide variety of planforms. Some account of this work is contained in the book of Jones and Cohen[31] and the article of Heaslet and Lomax.[32] Only a very limited amount of work for high angles of attack was carried out.

I became interested in the subject of supersonic wing-body interference in the late 1940's and set myself the task for a doctoral thesis of finding an exact solution for supersonic flow past a rectangular wing mounted centrally on a round body. I succeeded in finding an exact solution, but it came out in terms of new and untabulated functions.[33] The functions were particularly obdurate and defied accurate tabulation. I managed to get together tabulations adequate for the purpose of the thesis, but more extensive and accurate tabulations were made thereafter.[34] The success of the subsequent tabulations was due in no small measure to the mathematical skills of Dr. William Mersman of NASA Ames Research Center. With the computing machines now available, the same wing-body interference calculation by panel methods is quick and easy.

Subsequent to the foregoing mathematical investigation of supersonic wing-body interference, my fellow workers and I at Ames Research Center developed an approximate engineering method of treating wing-body interference at subsonic and supersonic speeds.[35] The method has been used for the preliminary design of both missiles and airplanes.

My remarks constitute only a short review of the large amount of work carried out in linearized supersonic theory by many investigators in the decade or more following World War II. It was a rewarding field because useful engineering results could be obtained, usually in closed form, without the use of computers. However, such results are limited to small angles of attack because of the viscous effects occurring at high angles of attack. There are also limitations at transonic speeds and hypersonic speeds because of nonlinear effects of compressibility.

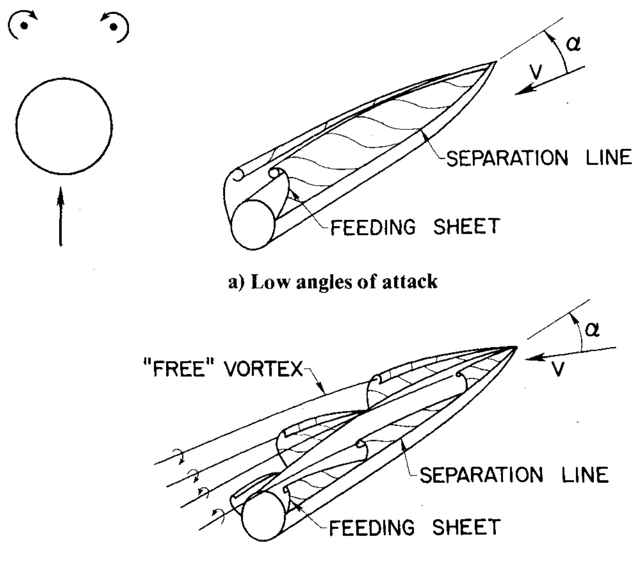
Effects of Viscosity and Vortices

While both wings and bodies encounter aerodynamic nonlinearities due to viscous effects, particularly at high angles of attack, engineering methods for dealing with these viscous effects were developed principally for bodies* before 1958. It has been realized for some time that slender bodies of revolution at moderate angles of attack develop symmetric vortices on their leeward side (Fig. 4a). In a cross-flow plane the flow resembles a well-known problem in potential flow: that of a symmetrical pair of point vortices in the wake of a cylinder shown in the inset of Fig. 4. It was thus possible to associate the three-dimensional viscous flow of Fig. 4a with the above two-dimensional inviscid flow in which all vorticity is concentrated at the vortex centers. Many theoretical efforts to explain the three-dimensional solution were attempted using various boundary conditions to determine vortex strengths and positions. It is interesting that a 1913 solution by Föppl[36] has been used in this connection. A later more sophisticated solution by Bryson[37] has also been used to obtain nonlinear loads on inclined slender bodies. These attempts follow a recurring theme in theoretical aerodynamics of trying to model a viscous flow using a potential flow with concentrated vortices in a fashion to resemble the real flow.

A further extension of the theory for the foregoing problem was made by means of an unsteady two-dimensional analogy. Consider a plane normal to the body axis and fixed in the fluid through which a body of revolution at angle of attack passes. The body cross section both changes radius and moves vertically in this plane in a two-dimensional sense. The analogy with slender-body theory is evident. If one knows the two-dimensional drag coefficient of such a two-dimensional flow, then one can estimate the normal force per unit axial length on the body for the three-dimensional steady flow. Since the cross-flow drag of a cylinder varies with the square of the flow speed normal to the cylinder, a quadratic dependence on angle of attack of body normal force due to the vortices was found. Numerous implementations of this general model were developed, of which that of Allen and Perkins[38] is particularly well known and widely used in missile aerodynamics.

Recently it has been widely appreciated that at high angles of attack, a body of revolution can develop a side force either to the left or to the right, or a side force which switches randomly from left to right. This phenomenon, which has

*The first nonlinear theory for wings of low aspect ratio is that of W. Bollay (*Journal of Aeronautical Sciences,* Vol. 4, 1937, p. 7). He assumed continuous vortex shedding from the side edges of a narrow rectangular wing and obtained a normal force proportional to the square of the angle of attack.



a) Low angles of attack

b) Moderate angles of attack

Fig. 4 Vortex formation on inclined body of revolution.

been termed "phantom yaw," was discovered and properly explained, probably for the first time, by Cooper, Gapcynski, and Hasel in 1952.[39] The cause of the side force is asymmetric flow separation resulting in an asymmetric wake over the leeward side of the body (Fig. 4b) which in a cross-flow plane resembles the structure in the near wake of the well-known Von Kármán vortex street. The vortex pattern appears to switch bistably between left-handed and right-handed mirror configurations.

Another area in which significant work was performed is in determining the base pressure of bodies with blunt bases. Chapman and associates[40] carried out extensive research in this subject. The powerful flow visualization means of the vapor screen was also introduced by Allen and his associates[38] in this period.

In summary, by 1958 a number of significant advances had been made in missile aerodynamics. Slender-body theory had been widely applied to missile aerodynamic problems. Likewise, supersonic theory for wings and bodies had been extensively worked out. Much attention had been given to wing-body and wing-tail interference effects and body viscous effects. However, very little attention had been given to high angles of attack and the extensive use of numerical methods was awaiting the debut of modern computers.

Present

The period from 1958 until now has seen a number of advances in missile aerodynamics, especially in the area of nonlinearities. In fact missile aerodynamics is now largely concerned with nonlinearities. The nonlinear phenomena producing missile forces (except drag), moments, or pressure coefficients which are not linear functions of angle of attack, angle of sideslip, or control deflection angle.

Nonlinear phenomena can be classified as inviscid (irrotational or rotational) and viscous. Probably the most significant inviscid effect is compressibility. Important viscous effects are usually associated with boundary-layer separation and/or vortices. Frequently the effects of compressibility and viscosity are combined. Nonlinear problems are well suited to the computer since analytical solutions are generally difficult or impossible. Accordingly, in the period since 1958 we see the introduction of the computer in the solution of missile aerodynamic problems.

Let us consider the present state-of-the art in missile aerodynamics under the categories of bodies, wings, wing-body combinations, and wing-body-tail combinations.

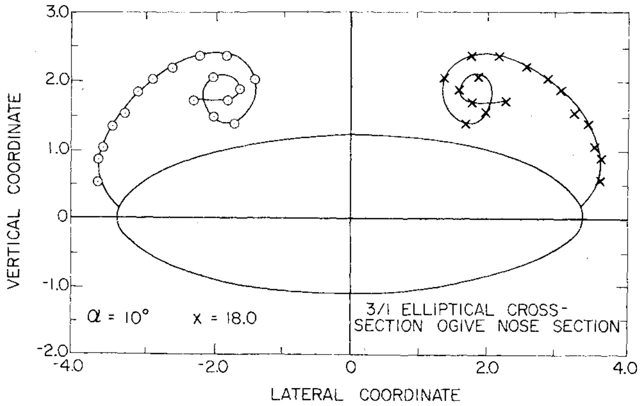


Fig. 5 Calculated vortex clouds for ogive nose of elliptical cross section.

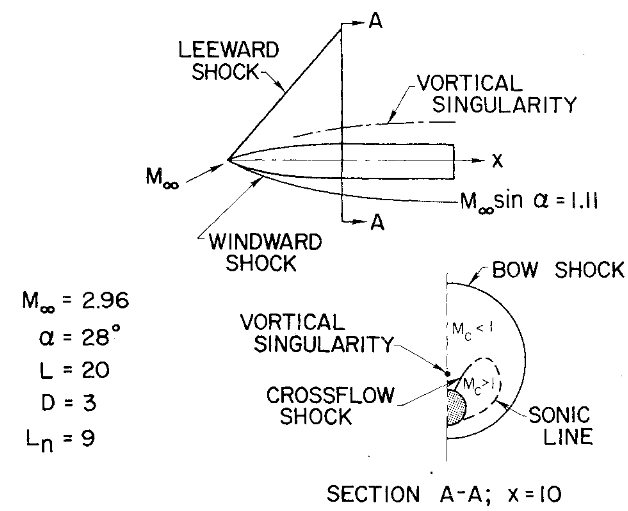


Fig. 6 Inviscid supersonic cross-flow about body at high angle of attack.

Bodies Alone

It is well known that for bodies of revolution with subcritical cross-flow Mach numbers ($M_c \leq 0.4$) there are several distinct patterns of the flow as the angle of attack increases. Figure 4 illustrates two of these patterns. At the lowest angle of attack, symmetric vortex shedding of the type treated by Allen and Perkins[38] occurs. In a higher range of angle of attack, asymmetric vortex shedding occurs. For this case the vortices, viewed in a plane normal to the body axis, appears steady with two or more vortices. Near the nose, only two vortices would appear. Further increase in the angle of attack can produce unsteady wake flow behind the cylinder together with buffeting. At 90 deg angle of attack the flow will be similar to that about a two-dimensional cylinder. It is for the first two types of flow that significant progress has been made since 1958.

For the symmetric vortex pattern, Allen and Perkins developed a method for determining the body axial loading distribution based on cross-flow drag methods. Jorgensen and Perkins[41] obtained the body vortex strengths and positions principally from experiment. Advances have been made in determining the flowfield and body pressure distributions. Wardlaw's[42] approach to the problem was to consider incompressible separated flow in a cross-flow plane and to calculate the vorticity shed per unit axial length at the separation line using experimental data for the separation line location. He used a marching procedure down the body and obtained a large number of vortices in the cross-flow plane. Marshall and Deffenbaugh[43] eliminated the necessity of using experimental separation-line data by performing a boundary-layer analysis of the cross-flow boundary layer. They were able to obtain asymmetric vortex patterns using their approach. Mendenhall and Nielsen[44] extended the Marshall-Deffenbaugh method to include noncircular bodies. An example case is shown in Fig. 5. The models all allow distributed vorticity in the body wake as opposed to concentrated vorticity as given by methods such as those of Foppl[36] and Bryson.[37]

For the most part the foregoing methods are incompressible, and were made possible through the use of large-scale computers. For inviscid, rotational, compressible flow a number of codes based on the Euler equations have been developed. One of these codes due to Pulliam and Steger[45] has been applied to determine the flowfield of a body of revolution at high angles of attack at supersonic speed. The calculated results are shown in Fig. 6.

Detailed cross-flowfields[46] for incompressible speeds have been measured using laser anemometry to check theory[44] with good results. The growing use of laser anemometry is a new experimental trend during this period.

When the Mach number of the onset flow in the cross-flow plane is supercritical, it is known[47] that the nature of the flow on the leeward side of a body is radically changed from that for subcritical cross-flow. In fact Chapman, Keener, and Malcolm[48] developed a rule of thumb that no significant side force exists on bodies of revolution if $M_c \geq 0.5$. The flow on the leeward side now exhibits large symmetric areas of circulating flow as shown in Fig. 7. Asymmetric vortices can emerge from the top of these areas. The vapor-screen pictures of Landrum and Babb[49] illustrate this phenomenon. No solutions for such flows are available. Some flowfield measurements with supercritical cross-flow have been made by Oberkampf[50] using conventional probes.

One area which is receiving attention is means for alleviating the effect of asymmetric flow separation in causing large side forces. These means are important in the transonic Mach number range since the conditions for the existence of asymmetric vortices generally occur only for freestream Mach numbers less than 1.2. These conditions are that the angle of attack be large, above approximately 25-30 deg, and that the cross-flow Mach number be less than about 0.5-0.6. It is too early to tell what the practical outcome will be of the methods currently being investigated to alleviate asymmetric vortex effects.

It is clear that while much progress has been made in body-alone aerodynamics since 1958, much remains to be done both theoretically and experimentally.

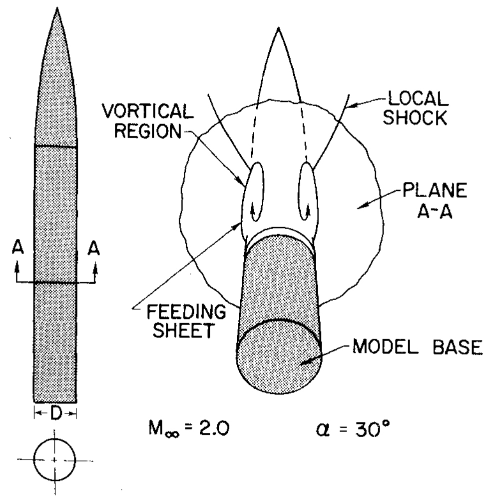


Fig. 7 Supercritical cross-flow in plane normal to axis of body of revolution.

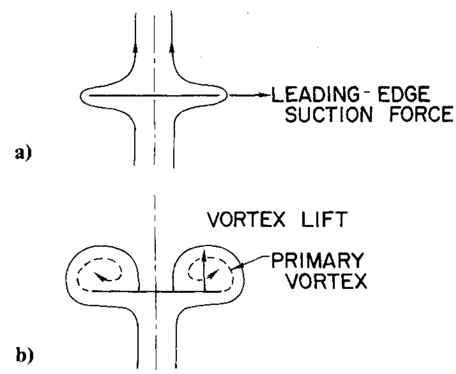


Fig. 8 Simplified sketch of streamlines in cross-flow plane of delta wing a) with and b) without leading-edge suction.

Wings Alone

Since 1958 a number of advances have been made in wing-alone aerodynamics. Aside from the development of the supercritical wing by Whitcomb, advances include the Polhamus vortex-lift analogy, insight into vortex bursting, and the use of advanced computer methods. However, the advances have not been sufficient to permit the calculation of wing-alone aerodynamic characteristics on a general basis for large angles of attack.

Missiles tend to use wings of lower aspect ratio than those of aircraft. One of the important properties of such wings is that a significant fraction of their normal force is proportional to the square of the angle of attack. Since low-aspect-ratio wings are body-like, it is not surprising that they should have a strong quadratic dependence of normal force on angle of attack which can be described by a cross-flow drag coefficient in a fashion similar to that for a slender body of revolution. Polhamus[51] has advanced the vortex-lift analogy as the source of the quadratic lift, a subject to which we now turn.

Consider the flow around a sharp leading edge of a delta wing at incompressible speed in a plane normal to the flow. Two types of flow are shown in Fig. 8. The first flow is the inviscid potential flow which can turn 180 deg around the sharp edge. In this case an in-plane suction force exists normal to the leading edge which tends to reduce the induced drag. However, because of viscosity, the flow cannot negotiate a 180 deg turn, and it tends to separate from the leading edge as shown in Fig. 8b. Reattachment occurs on the top surface with the formation of a leading-edge vortex. The low pressure in the vortex causes an additional normal force. Now for sharp leading edges, the vortex lift is approximately equal to the leading-edge suction. This is the Polhamus vortex-lift analogy.[51] The lower the aspect ratio, the larger the fraction of the total lift that is vortex lift. Low-aspect-ratio leading-edge extensions or strakes work on this principle, forming strong vortices which pass over the wings. The vortices also seem to inhibit root stall on the wing in a way which cannot be predicted as yet.

As the wing aspect ratio increases, the fraction of leading-edge suction converted to vortex lift for sharp leading edges decreases. The use of vortex lift to increase missile performance can be important, but methods for predicting the fraction of vortex lift to be realized practically need further development. Eventually viscous consideration such as boundary-layer theory must be introduced into the wing-alone model if rational predictions are to be made. One of the things that limits the amount of vortex lift that can be developed is vortex bursting over the wing. Vortex bursting appears to be an instability which causes a sudden increase in the diameter of a vortex accompanied with turbulence. A number of theoretical treatments of the subject exist, of which that of Mager[52] is well known. Engineering methods such as that of Wilson[53] exist for determining the vortex bursting position over delta wings. For low-aspect-ratio wings the angle of attack for vortex bursting can be as high as 40 deg.

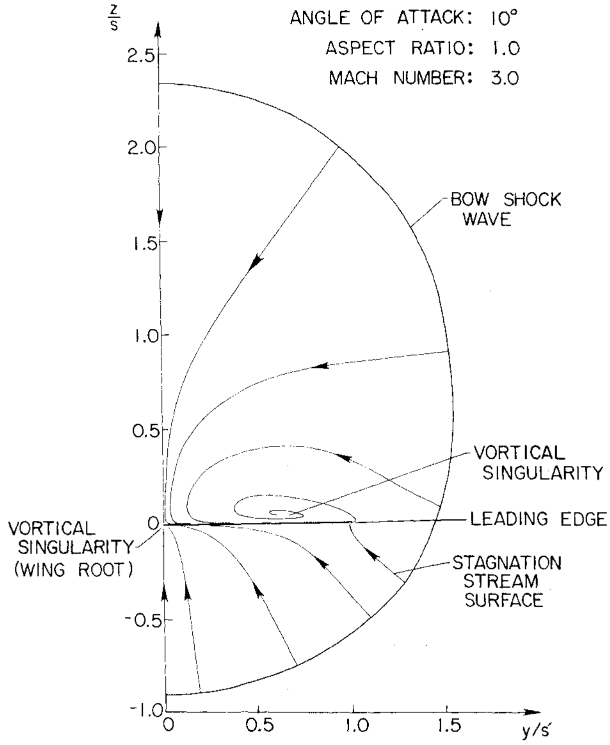
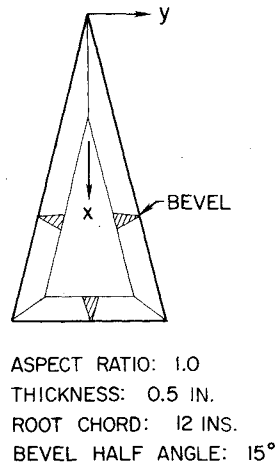
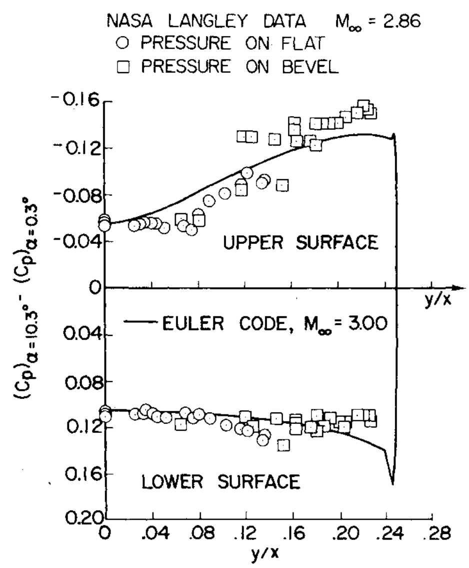


Fig. 9 Projection of streamlines on cross-flow plane as determined from Euler code with Kutta condition at leading edge of delta wing.



a) Pressure wing geometry



b) Upper and lower surface pressure distributions at $\alpha = 10$ deg

Fig. 10 Comparison of delta wing pressure distributions as measured and as calculated using an Euler code with Kutta condition.

A number of investigators have tried to develop methods based on potential theory for estimating the aerodynamic characteristics of wings with sharp leading edges. The methods generally replace the vortex cores and their feeding sheets with potential point vortices and solve the problem invoking various boundary conditions. Computer methods using the Euler equations or the Navier-Stokes equations will probably replace these methods.

An initial attack on the problem of determining the flowfield of a wing with sharp subsonic leading edges has been attempted by Klopfer and Nielsen[54] using the Euler equations. Potential theory is irrotational and inviscid, and does not include the possibility of continuous vorticity in the field. The Euler equations, on the other hand, are rotational and inviscid and thus include the capability of handling compressibility and continuous vorticity. If a boundary condition is imposed at the subsonic leading edge of a wing that the flow must leave the edge laterally with no vertical velocity, a flowfield similar to that shown in Fig. 8 might be expected. Some preliminary results calculated on this basis for an $\mathcal{R} = 1$ delta wing at $M_\infty = 3$ are shown in Fig. 9. The wing geometry is given in Fig. 10a and the calculated flowfield in Fig. 9 neglecting wing thickness.

The calculated flow is conical; that is, it has constant velocity components along any ray from the apex. Under these special circumstances the streamlines can be projected along radial lines through the wing apex onto a sphere to form a two-dimensional nonintersecting pattern. The streamline pattern on the sphere can in turn be projected onto a cross-flow plane. Figure 9 was constructed in this fashion. The streamlines entering the bow shock wave on the lower surface generally feed into a "vortex singularity" on the windward plane of symmetry. A streamline entering the bow shock wave from the right comes to a cross-flow stagnation point on the lower surface just inboard of the leading edge. There is a large vortical region above the wing with a vortex singularity in its center into which the streamlines above the stagnation streamline feed. The wing tip has become a source of vorticity which feeds the circulating region.

The calculated wing pressure field corresponding to the flow of Fig. 9 is conical. Unpublished pressure data for a triangular wing of aspect ratio unity is available for $M_\infty = 2.86$. These data are compared with the Euler code prediction in Fig. 10b. I am indebted to Robert Stallings of NASA Langley Research Center for permission to use the unpublished data. The theoretical and computational work was carried out under the sponsorship of the Office of Naval Research. The computational work was supported jointly by ONR and NASA/Ames. Permission to use this unpublished work was kindly given by David Siegel of ONR.

A Russian study of leading-edge separation using Euler equations is to be found in Ref. 55.

The calculation of wing characteristics at transonic speeds for both steady and unsteady flow, as exemplified by the work of Ballhaus, Bailey, and Frick,[56] has been much advanced in recent years through high-speed computers. While results for low angles of attack have been gratifying, the efforts in this area are still in an early stage of development for high angles of attack. Theoretically the efforts are limited by the lack of understanding of turbulent and separated flows which occur on wings at high angles of attack. Experimentally, advances have been limited by problems of supporting the wings in the wind tunnel while at the same time keeping support interference sufficiently small.

Wing-Body Interference

The subject of wing-body interference is a particularly important one in missile aerodynamics because body diameter can be a large fraction of the total wing span. Under these conditions the interference of the body on the wing panels (monoplane or cruciform) and of the wing panels on the body can be large. It is convenient to consider the subject from the viewpoint of 1) methods for calculating detailed pressure distributions, 2) engineering methods for predicting component loads and center-of-pressure positions, and 3) special problem areas.

In the last decade panel methods for calculating the detailed pressure loading on complete aircraft have attained considerable prominence.[57,58] These methods, which generally solve potential equations and are valid within the linear range of angle of attack, are applicable to missiles. Special codes adapted to cruciform missiles have been constructed.[59] They have also been extended to include vortex-induced effects so that their validity extends beyond the range of linear theory.[60] In the transonic range, computer programs for wing-fuselage combinations have been developed,[61] but comparable codes for cruciform missiles await development. Generally speaking, the foregoing computer codes are not suitable for preliminary design because of long run times and because they are limited to low angles of attack.

Preliminary engineering and design methods for determining fin and body forces and moments for missiles at zero roll and in the linear range are basically no different than those for aircraft.[35,62] However, until recently[47] there has not been a generally available engineering method for cruciform missiles at arbitrary roll angle. The U.S. Navy has developed an engineering prediction program particularly suited to its needs which considers drag force in some detail.[63] Existing engineering prediction methods need further development in the areas of control effectiveness, hinge moments, and high-angle-of-attack effects.

Not much attention has been paid to engineering prediction methods for wing-body combinations utilizing bodies of noncircular cross sections with the possible exception of elliptical bodies. Jorgensen[64] has done a significant amount of work in this area particularly in the high-angle-of-attack range.

High-angle-of-attack effects are currently receiving much attention because increased missile maneuverability can be obtained at such angles. Such effects are principally concerned with nonlinearities. For instance, if the cross-flow Mach number is supersonic, there is shock structure on the windward side of the body which has an adverse effect on wing-body interference. A measure of the interference of the body on the wing is given by the parameter K_W, which is the ratio of the normal force on a fin (at zero roll) to that which the fin would develop at the same angle of attack as part of a wing alone formed by two fins. Figure 11, based on experimental data, shows how K_W varies with angle of attack and Mach number for a particular wing-body combination. These results are based on data with the body vortex effects subtracted out of the panel loads. Favorable interference, $K_W > 1$, occurs for lower angles of attack and Mach numbers. However, at the higher Mach numbers the wing-body interference can be quite unfavorable at high angles of attack. At the present time no theoretical method has been applied to the calculation of these effects.

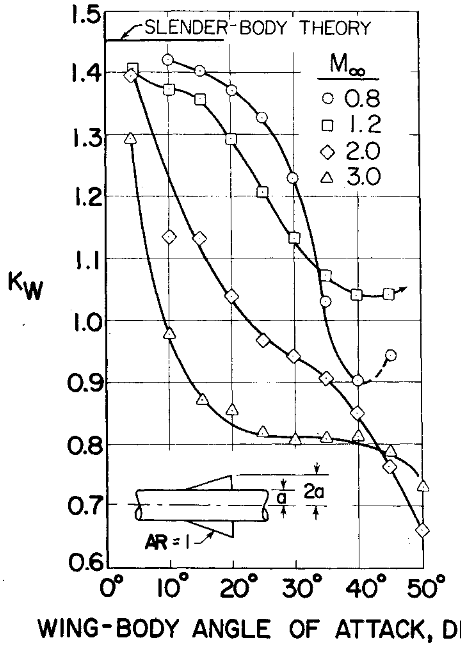


Fig. 11 Effect of angle of attack and Mach number on wing-body interference.

Wing-Body-Tail Configurations

Wing-body-tail configurations exhibit all the phenomena of bodies alone, wings alone, and wing-body or tail-body combinations. In addition they exhibit conventional wing-tail interference and special afterbody-tail interference effects which will be discussed.

Methods for calculating wing-tail interference used for aircraft (e.g. the method of Silverstein and Katzoff[5]) are generally applicable to missiles for zero roll angle. However, there is additional interference caused by nose vortices which may pass over the canard fins and the afterbody to influence the tail. Methods for including the effect of body and canard vortices on tail interference and of tracking them over the length of the missile to the tail have been developed which are applicable to a cruciform missile at any arbitrary roll angle.[47,65] Reverse-flow methods have been applied to the problems of calculating panel loads and center-of-pressure positions due to the vortices acting on cruciform empennages.

Wing-tail interference takes on a different character at high angles of attack if the length of the afterbody between canards and tail (in body diameters) is not small. In this case the nose and canard vortices pass sufficiently high above the tail to have small effect on it. However, the afterbody itself can develop cross-flow vortex clouds which pass close to the tail and dominate the tail interference. Figure 12 shows the vortex pattern in a cross-flow plane in front of the tail of an inline cruciform canard missile at 35 deg angle of attack. The canard vortices indicated by C_1, C_2, C_3, and C_4 in this figure are high enough above the empennage to have a minor influence on it with the possible exception of C_3. Clouds of clockwise and counterclockwise vortices have been shed by the afterbody, and these clouds are close to the tail fins T_1 and T_2, shown by dotted lines. This particular calculative example also shows that the vortex shed by the windward canard fin C_3 has passed upward close to the left-hand side of the body and has been captured by the left afterbody vortex cloud. At large angles of attack, the afterbody can have the dominant interference effect on the empennage, and a large side force, yawing moment, and rolling moment can result. At present, these interference effects are modeled in a gross way, and further work is required to understand them thoroughly and to predict them accurately.

Future

Let us turn now to the prediction of future developments in missile aerodynamics. This is a task which is simultaneously appealing and daunting; the former, because one is not constrained by facts, and the latter because of the ever-present danger of mis-prognostication with its subsequent effects upon future readers of this paper. Indeed, with future

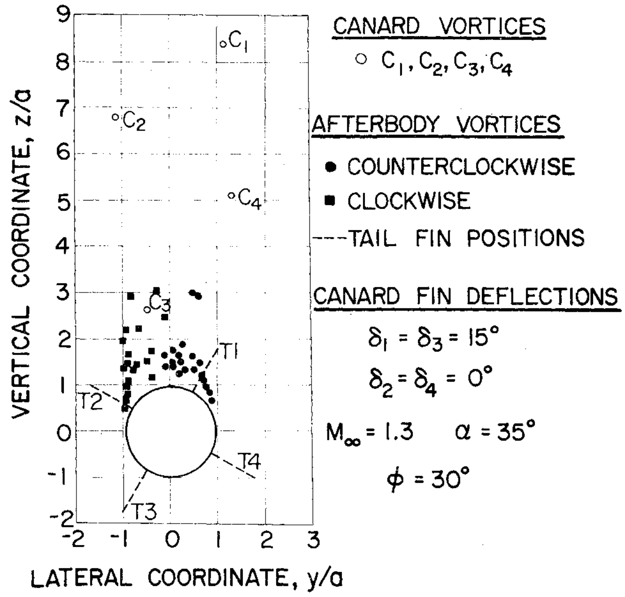


Fig. 12 Canard vortex location and afterbody vortex clouds at crossflow plane of tail section for cruciform canard missile.

developments in energy and laser weapons, these readers may be few and confined to archivists of technology. I am reminded of a prognostication made soon after the introduction of the railroads at the beginning of the 19th century. It was predicted that speeds would never exceed 20 mph because the human body could not withstand such high velocities! We can now smile, realizing that the problem is rather the inability of prognostication to withstand the velocity (changes in speed and direction) of technological progress.

For purposes of discussion, I have selected several important subjects in which progress should be made.

High-Angle-of-Attack Aerodynamic Theory

During the course of this paper much has been said of the status of high-angle-of-attack missile aerodynamics. Generally speaking, adequate predictive methods do not exist for determining the high-angle-of-attack aerodynamic characteristics of wings, bodies, wing-bodies, etc., including nonlinearities due to compressibility and viscosity. The present and future use of high angles of attack to obtain greater maneuverability and other advantages definitely makes it important to emphasize the development of such predictive methods in the future.

Cruciform vs Monoplane Wings

The aerodynamic advantages of cruciform (or triform) wings for producing normal forces in the pitch plane substantially independent of roll orientation has been an important factor in their past utilization. However, their use has been accompanied by certain difficulties. In the first place, a cruciform missile is basically a combination of four corner reflectors, a fact which tends to increase its radar cross section and its detectability. Second, since cruciform missiles generally roll, they are not well adapated to airbreathing propulsion and thus suffer a range penalty. Also at high angles of attack, the induced rolling moments can cause them to roll too fast for the control system to follow. Finally, rolling missiles imply round bodies so that possible advantages of noncircular bodies could not be pursued. The search for increased ranges, decreased detectability, simplified control systems, and increased aerodynamic efficiency may thus lead in the future to missiles that fly like airplanes. It seems that in missiles we are coming back to the Wright Brothers!

Integration of Missile Airframes and Engines

For missiles with solid fuel motors the plume behind the base has generally produced little aerodynamic interference on the airframe except for very underexpanded jets. For airbreathing engines, there can be significant mutual interference between the inlets and the airframe. Airbreathing engines are sensitive to inlet conditions which are functions of the inlet position on the airframe as well as missile operating conditions. It is difficult to design inlets of high pressure recovery for high angles of attack and Mach numbers, and efficient inlets for rolling missiles are also difficult to design. The whole problem of integrating missile airframes and airbreathing engines will receive much greater attention, particularly at high angles of attack and high Mach numbers. The application of innovative design concepts and computational fluid dynamics to these problems may be a trend of the future.

Integration of Aircraft and Missiles

In recent years the addition to the airframe of many missiles externally mounted on racks and pylons has resulted in large drag penalties to aircraft optimized for minimum drag without external stores. In fact, the missile installations have turned otherwise supersonic aircraft into subsonic ones. Up to the present time the clean separation of external stores from aircraft and the performance penalties due to hanging external stores on aircraft have been investigated principally in expensive wind tunnel and flight tests. The large number of combinations and permutations of aircraft and stores requires extensive testing. Recently the power of large-scale computers has been brought to bear on these problems. It is probable that computer analysis of these problems can profitably be greatly expanded. Also innovative ideas for integrating stores and airframe for improved performance will probably receive greater emphasis.

In the future it is to be expected that the design of the aircraft and its external stores will be considered as a combined problem.

High-Angle-of-Attack Measurements

The operation of missiles (and airplanes) to high angles of attack has brought about a need for improved methods for high-angle-of-attack testing. It is difficult to test models with supports which will stand high-angle-of-attack loads and at the same time minimize the effects of support interference on the loads. Much work is required in this area.

Since it is not generally possible to predict the aerodynamic characteristics of missiles at high angles of attack on a priori grounds, reliance must be placed on wind tunnel tests for some time in the future. Much systematic basic data needs to be obtained on wings, bodies, and wing-body combinations. Laser anemometry to measure flowfields is now in use, but improvements are required to get better coverage and speed up data acquisition. While special testing devices to obtain damping data at high angles of attack are being developed, they should receive more attention in the future.

Exploitation of Large-Scale Computers

The future role of large-scale computers in missile aerodynamics has been alluded to in many of the preceding sections. The fact of the matter is that the computer has not been exploited for missile aerodynamics to nearly the same extent as for airplane aerodynamics. A reversal of this trend has already started and should accelerate in the future. The computer provides an ideal tool for attacking the important problems of high-angle-of-attack aerodynamics.

A number of levels of sophistication in the use of the computer are clear in its application to missile aerodynamics. A level of low sophistication might be represented by engineering prediction methods based on approximate aerodynamic models together with empirical input data. A more sophisticated level might be represented by the application of the Euler equations to missile aerodynamics. It seems to me that the Euler equations can be successfully applied to a whole range of missile aerodynamic problems without necessitating the use of the Navier-Stokes equations. In the first place, the sources of vorticity such as leading edges, trailing edges, and separation lines can be handled using Euler equations by proper treatment of the boundary conditions as has been previously mentioned in connection with Figs. 9 and 10. In the second place, convection of the vorticity is properly handled by the Euler equations. In the third place, the dispersion of vorticity due to viscosity (laminar or turbulent), which is not handled by the Euler equations, is not significant for many problems in missile aerodynamics. A high degree of sophistication in the use of the computer is represented by the use of the Navier-Stokes equations with turbulent modeling.

With regard to "exact" Navier-Stokes solutions, the limiting factors in their use at the present time include the unavailability of machines big enough to handle three-dimensional flowfields, high computer costs, and the uncertainty in turbulence modeling. NASA is studying the problem of the speed and size of the computer required, and it could be a comparatively short time before such a machine is a reality. Historically, computational costs have been dropping at an exponential rate, but it seems to me that the Navier-Stokes solutions will not be used extensively in preliminary design for a long time. The value of such solutions may be principally for the validation of final designs and for evaluation of less sophisticated but faster prediction methods.

Turbulence modeling at the present time presents formidable difficulties. Promising progress is being made in this area by means of large-scale eddy simulation.[66] However, the question of how good a turbulence model must be has not been answered. The required accuracy of the turbulent model will probably depend on the application at hand, including the configuration as well as the physical quantity of interest. It is conceivable that a large number of turbulence models will be developed for different applications, and that a data bank of such models will be developed. In this connection the computer can lift itself by its own bootstraps since it can be used as an experimental tool in turbulence modeling. It is my opinion that the question of a national data bank of turbulence models should be addressed by NASA.

I cannot conclude a discussion of large-scale computers without addressing the question of the future role of the wind tunnel vs the computer. Much controversy has surrounded the subject since the thought-provoking paper of Chapman, Mark, and Pirtle.[67] In their paper they state: "When a sufficiently advanced computer becomes available, we believe it will displace the wind tunnel as the principal facility for providing aerodynamic flow simulation." There is no doubt in my mind that many measurements now made in the wind tunnel can be calculated just as well on large computers, and that more of the conventional wind tunnel problems will be tractable on computers in the future. The rate at which this will happen can be argued. However, the above quote does not imply that the wind tunnel will be superseded by the computer. Indeed, it can be argued that the requirements for wind tunnels will be increased. The wind tunnel can reproduce fluid mechanical phenomena for which the physics is not understood and hence which cannot be put into a computer. Also, wind tunnels and computers can be used to verify the results of one another. Wind tunnels and computers can reinforce each other in other synergistic ways in such applications as "smart" wind tunnels and conditional sampling. The requirements for both will thus continue and, in my view, will increase.

Other Topics

Several other topics will now be suggested as likely candidates for future developments in missile aerodynamics: 1) strake aerodynamics, 2) blended wing-body combinations, 3) controls, and 4) asymmetric vortex alleviation.

Strakes have produced improved aerodynamic performance in aircraft in preventing wing stall and providing good aerodynamic characteristics to high angles of attack. A rational theory of strake design remains to be developed. It should take account of strake-body interference.

The use of wing-body combinations wherein the wing blends into the body without sharp curves could be a fruitful line of future investigations from the standpoints of aerodynamic efficiency and radar detectability. The solutions of optimal problems in this area would be of interest. Noncircular bodies will receive greater emphasis.

Not much has been said concerning aerodynamic or other types of controls. There is presently no means other than lengthy wind tunnel experiments for determining the nonlinear control characteristics that are found at high angles of attack, particularly in the transonic range. This very important subject has not received the attention it deserves.

The phenomenon of "phantom yaw" induced by asymmetric body vortex separation is currently receiving much attention. Novel means for controlling the size and magnitude of the asymmetry or of eliminating it altogether are needed.

Concluding Remarks

The author wishes to acknowledge the honor the AIAA has bestowed on him by inviting him to present this Wright Brothers Lecture. My purposes have been to give visibility to the aeronautical community generally of the importance of missile aerodynamics and to suggest specific areas which need future attention. If I have achieved these goals, I will be most pleased.

Acknowledgments

I should like to acknowledge the efforts of my colleagues, Selden Spangler, Michael Hemsch, and John Fidler, who have made many helpful suggestions and who have reviewed the text. I am also indebted to Professors John R. Spreiter and Krishamurti Karamcheti of Stanford University, who have reviewed the text and made a number of valuable suggestions.

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